

Effect of loops on spiral lattice animal statistics

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

1988 J. Phys. A: Math. Gen. 21 L979

(<http://iopscience.iop.org/0305-4470/21/20/004>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 129.252.86.83

The article was downloaded on 01/06/2010 at 06:02

Please note that [terms and conditions apply](#).

LETTER TO THE EDITOR

Effect of loops on spiral lattice animal statistics

Indrani Bose†, Purusattam Ray‡ and Subhashish Mukhopadhyay§

† Department of Physics, Bose Institute, 93/1 Acharya Prafulla Chandra Road, Calcutta 700 009, India

‡ Saha Institute of Nuclear Physics, 92 Acharya Prafulla Chandra Road, Calcutta 700 009, India

§ Computer Centre, University of Burdwan, West Bengal, India

Received 22 July 1988

Abstract. We have shown by exact enumeration on a square lattice that spiral lattice site animals and spiral lattice site trees belong to different universality classes, i.e. loops have a non-trivial effect on spiral lattice animal statistics. This result is contrary to what is known in the cases of undirected and directed lattice animals. Some special features of spiral lattice animals have been pointed out in this context.

Lattice animals, defined as clusters of connected sites or bonds embedded in a regular lattice, are lattice models of branched polymers in dilute solution and also describe the statistics of large clusters below the percolation threshold (Lubensky and Isaacson 1979, Stauffer 1979, 1981). Two types of lattice animals that have been extensively studied are the undirected and directed lattice animals (Stauffer 1979, 1981, Dhar 1986). Undirected lattice animals can be constructed without any constraint on the cluster connectivity whereas in the case of directed lattice animals sites or bonds can be occupied only in certain specific lattice directions. Recently, a new type of lattice animals, known as spiral lattice animals, have been defined (Li and Zhou 1985, Bose and Ray 1987, Bose *et al* 1988) which belong to a new universality class, different from those of undirected and directed lattice animals. In a spiral animal the constraint is such that each site of the cluster is attached to the origin through at least one spiral path. In the spiral path, connection is either in the forward direction or in a specific rotational direction, say, clockwise. An interesting aspect of lattice animal studies has been finding the effect, if any, of loops on the animal statistics. Work carried out so far (Lubensky and Isaacson 1979, Family 1980, Daoud and Joanny 1981, Gaunt *et al* 1982, Duarte 1985) indicates that for both undirected and directed lattice animals the number of loops is a non-critical quantity, i.e. animals with loops and animals without loops (trees) belong to the same universality class. In this letter, we study the effect of loops on the statistics of spiral lattice animals. The study is based on exact enumeration of spiral lattice site animals and trees on a square lattice for size S up to 12 (animals) and 13 (trees) sites.

In describing the effect of loops on spiral lattice animal statistics, we will concentrate on the following two asymptotic relations. The first relation states that in the asymptotic $S \rightarrow \infty$ limit, the total number of animals g_S goes as

$$g_S \underset{S \rightarrow \infty}{\sim} \lambda^S S^{-\theta} \quad (1)$$

where λ is a constant for a particular lattice and is known as the 'growth parameter' because asymptotically $g_S/g_{S-1} \rightarrow \lambda$. The animal number exponent θ is universal and changes only if lattice dimension d changes. The second relation shows how the average radius of gyration R_S scales with size S :

$$R_S \underset{S \rightarrow \infty}{\sim} S^\nu \quad (2)$$

where ν is the radius of gyration exponent. R_S is defined as

$$R_S = \left\langle \left(\sum_{i=1}^S r_i^2 / S \right)^{1/2} \right\rangle \quad (3)$$

where r_i is the distance of a cluster site i from the centre of mass of the cluster and $\langle \dots \rangle$ denotes the average over all animals. In order to calculate the exponents θ , ν and the parameter λ for both spiral animals and spiral trees exact enumeration of animals and trees has been performed. The computer algorithm used generates undirected animals and then checks each animal for spiral connection. Besides trees (no-loop animals) one-loop animals have also been separately counted. All these enumeration data, along with the exact data for radii of gyration, are listed in table 1. The number of loops L in an animal has been determined through the following relations:

$$b = \frac{1}{2} \sum_{i=1}^4 in(i) \quad L = b - S + 1 \quad (4)$$

where b is the number of bonds in the animal and $n(i)$ is the number of sites in the animal which have i nearest-neighbour occupied sites. Figure 1 shows the variation of the ratio g_S/g_{S-1} with $1/S$ for both animals and trees ($L=0$). Extrapolation of the last few points to $S \rightarrow \infty$ can be performed employing a standard numerical procedure (Gaunt and Guttmann 1974). The intercept on the y axis gives λ and the exponent θ is determined from the slope. For the radius of gyration data, we used a sequential fit to the form

$$\log R_S = \nu \log(S + \delta) + B + C/(S + \delta)^2$$

Table 1. Exact data on square lattice spiral animals, trees and one-loop animals.

S	Animals		Trees		One-loop animals	
	g_S	R_S	g_S	R_S	g_S	R_S
1	1	0.00	1	0.00	0	0.00
2	2	0.50	2	0.50	0	0.00
3	6	0.716 66	6	0.716 66	0	0.00
4	17	0.910 29	16	0.923	1	0.707
5	49	1.078 04	41	1.113 878	8	0.894 37
6	140	1.231 843	104	1.291 327	34	1.066 03
7	396	1.374 336	262	1.454 111	112	1.242 45
8	1 114	1.506 745	648	1.608 09	344	1.412 91
9	3 134	1.629 771	1 577	1.756 12	1 032	1.568 15
10	8 824	1.744 838	3 790	1.898 981	2 986	1.712 813
11	24 924	1.852 868	9 014	2.037 25	8 384	1.850 76
12	70 618	1.954 931	21 242	2.171 439	23 011	1.983 384
13	—	—	49 676	2.301 906	61 924	2.111 801

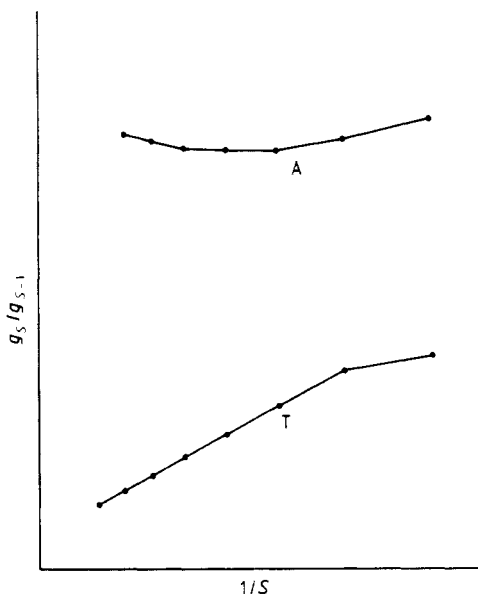


Figure 1. Plot of the ratios g_S/g_{S-1} against $1/S$ for spiral animals (A) and trees (T) on a square lattice.

the parameter δ being chosen in the range of values which gives a satisfactory convergence of the successive estimates of ν . The values of λ , θ , ν are

$$\lambda = 2.992 \pm 0.01 \quad \theta = 0.365 \pm 0.05 \quad \nu = 0.505 \pm 0.02$$

for animals and

$$\lambda_0 = 2.124 \pm 0.003 \quad \theta_0 = -1.315 \pm 0.02 \quad \nu_0 = 0.66 \pm 0.02$$

for trees. It is clear from the above results that spiral lattice animals and spiral lattice trees belong to different universality classes, i.e. unlike in the cases of undirected and directed lattice animals, the number of loops is a critical quantity and loops have a non-trivial effect on spiral lattice animal statistics. For both undirected and directed lattice animals, a number of studies (Whittington *et al* 1983, Duarte 1985, Lam 1987) have established that the growth parameter λ_L and the radius of gyration exponent ν_L for animals with L loops are equal to λ_0 and ν_0 , the growth parameter and radius of gyration exponent respectively for trees. Also, the animal number exponent $\theta_L = \theta_0 - L$. For spiral one-loop animals, $\lambda_1 = 2.05 \pm 0.1$, $\theta_1 = -4.072\ 398\ 94 \pm 1$ and $\nu_1 = 0.67 \pm 0.04$. Within the limits of error it appears that $\lambda_1 = \lambda_0$, $\nu_1 = \nu_0$ but $\theta_1 \neq \theta_0 - 1$. A longer series of enumeration data than that for animal size $S = 13$ is required for both trees and L -loop animals in order to find the relation, if any, between the exponents θ_L and θ_0 .

We have seen that loops have a non-trivial effect on asymptotic cluster properties of spiral lattice animals. Two features of the spiral animal structure should be noted in this context. Figure 2 shows a spiral lattice site animal containing three loops. The origin is denoted by a cross and the path traced out by arrows indicates cluster connectivity. The path goes either straight or turns clockwise. As soon as the path deviates from a certain direction it has to trace out a loop in order to continue growing in the original direction. To grow in all directions, the spiral animal has to contain a large number of loops. This is not true in the case of undirected and directed lattice

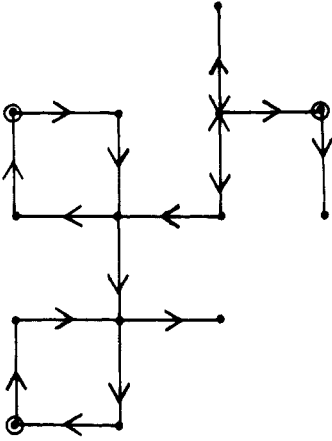


Figure 2. A spiral lattice site animal on a square lattice grown from the origin X . The animal contains three loops.

animals. The second feature has to do with the fact that not all trees, obtained from spiral lattice animals by eliminating appropriate occupied sites or bonds, are spirally connected. In figure 2, if the circled sites are not occupied sites, the lattice animal is a tree. This tree, however, is no longer spirally connected. In the case of directed lattice animals a similar procedure gives rise to trees which always obey the directional constraint.

To the authors' knowledge, spiral lattice animals are the only type of lattice animals known at present for which the number of loops is a critical quantity in the asymptotic limit. We have given numerical evidence to establish this fact. Further studies, including a possible field theoretic formulation of the problem, should be attempted to have a clear understanding of the effect of a spiralling constraint on the statistics of lattice animals.

References

- Bose I and Ray P 1987 *Phys. Rev. B* **35** 2071
 Bose I, Ray P and Dhar D 1988 *J. Phys. A: Math. Gen.* **21** L219
 Daoud M and Joanny J F 1981 *J. Physique* **42** 1359
 Dhar D 1986 *Physica* **140A** 210
 Duarte J A M S 1985 *J. Physique Lett.* **46** L523
 Family F 1980 *J. Phys. A: Math. Gen.* **13** L325
 Gaunt D S and Guttmann A J 1974 *Phase Transitions and Critical Phenomena* vol 3, ed C Domb and M S Green (New York: Academic) p 97
 Gaunt D S, Sykes M F, Torrie G M and Whittington S G 1982 *J. Phys. A: Math. Gen.* **15** 3209
 Lam P M 1987 *Phys. Rev. A* **35** 349
 Li T C and Zhou Z C 1985 *J. Phys. A: Math. Gen.* **18** 67
 Lubensky T C and Isaacson J 1979 *Phys. Rev. A* **20** 2130
 Stauffer D 1979 *Phys. Rep.* **54** 1
 ——— 1981 *Disordered Systems and Localization* ed C Di Castro and L Peliti (Berlin: Springer)
 Whittington S G, Torrie G M and Gaunt D S 1983 *J. Phys. A: Math. Gen.* **16** 1695